4E4161

Roll No. ________ Total No. of Pages : 6

4E4161

B. Tech. IV Sem. (Main/Back) Exam; April-May 2017

Computer Science

4CS2A Discrete Mathematical Structure

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 26

Instructions to Candidates :-

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*
Units of quantities used / calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL **2.** NIL

UNIT - I

1 (i) Define power set. If S be a finite set of order n then prove that power set $P(S)$ is a finite set of order 2^n.

2+6=8

(ii) Define the following :

(a) Cross partition of a set.

(b) Duality

(c) Floor function or greatest integer function.

(d) Bijection.

2×4=8

OR

2 (a) Show that the set of odd positive integers is a countable set. 4

(b) A survey is taken on method of commuter travel. Each respondent is asked to check BUS, TRAIN or AUTOMOBILE as a major method of travelling to work. More than one answer is permitted. The results reported were as follows :

(i) 30 people checked BUS;

(ii) 35 people checked TRAIN;

(iii) 100 people checked AUTOMOBILE;

(iv) 15 people checked BUS and TRAIN;

(v) 15 people checked BUS and AUTOMOBILE;

(vi) 20 people checked TRAIN and AUTOMOBILE;

(vii) 5 people checked all three methods.

How many respondents completed their surveys ? 4

(c) State and prove the generalized pigeonhole principle.

2+6=8

UNIT - II

2 (i) Define :

(a) Boolean matrix

(b) Product of Boolean matrices

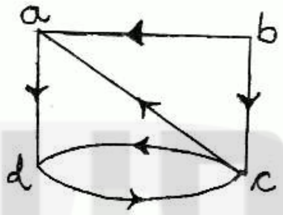
(c) Join and meet of Boolean matrices.

Also compute the join and meet of matrices :

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

1×4+4=8

(ii) Let R be the relation with digraph shown below. Find the transitive closure of R using Warshall's algorithm.



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OR

2 (i) Define congruency relation in Modulo system. If $A = Z$ (the set of integers), Relation R defined in A set by aRb as "a is congruent to b mod 2", then prove that R is an equivalence relation.

2+6

(ii) If the set of integers $I = \{......, -3, -2, -1, 0, 1, 2, 3, ...\}$ be partitioned by the equivalence relation aRb as $a \equiv b \pmod 3$. Obtain the set I/R.

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(iii) If $A = \{1, 2, 3, 4, 12\}$, the partial order of divisibility on A is $a \le b$ (i.e. if a divides b). Then draw the digraph and Hasse diagram of the poset $(A, \le)$.

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UNIT - III

3 (i) Prove by mathematical Induction that $3^n > n^3$ for all integers $n \ge 4$.

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(ii) Prove the implication "If n is an integer not divisible by 3, then $n^2 \equiv 1 \pmod 3$".

8

OR

3 (i) Write short notes on :

(a) Vacuous proof

(b) Trivial proof

(c) Constructive proof

(d) Non-constructive proof

1.5×4=6

(ii) Prove that the linear search algorithm works correctly for every $n \geq 0$.

4

(iii) Sort the list $X = [64, 25, 12, 22, 11]$ using selection sort algorithm.

6

UNIT - IV

4 (i) Sketch the complete graphs k_n, $1 \leq n \leq 6$.

1×6=6

(ii) Show that the complete digraph with n-nodes has the maximum number of edges i.e. $n(n-1)$ edges, assuming there are no loops.

6

(iii) Draw graph which is Eulerian as well as Hamiltonian.

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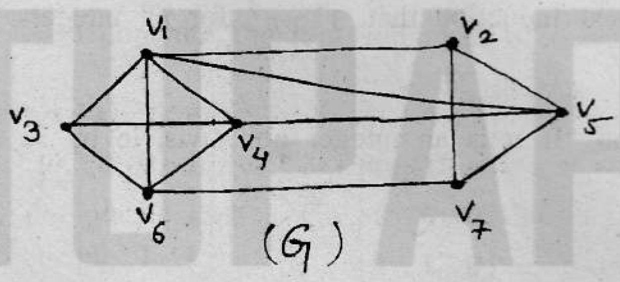
OR

4 (i) Use Welch-Powell algorithm to paint the following graph with minimum number of colors.



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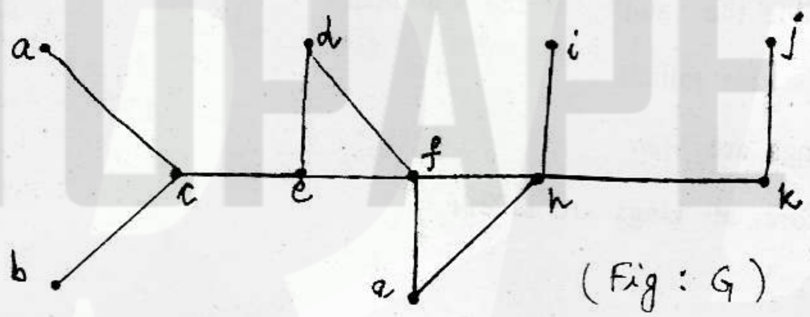
(ii) Prove that the chromatic number of a graph will not exceed by more than one, the maximum degree of the vertices in a graph. **6**

(iii) Use Depth-first search to find a spanning tree for the following graph G.



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UNIT - V

5 (i) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. **4**

(ii) Find PCNF of a statement S whose $PDNF$ is
$(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$. **6**

(iii) Is the following argument valid ?
Dhruv, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high - paying job. Therefore, someone in this class can get a high paying job. **6**

OR

5 (i) Define Tautology, contradiction and contingency. Determine the contrapositive of each statement :

(a) If John is a poet, then he is poor.

(b) Only if Mary studies will she pass the test.

$2 \times 4 = 8$

(ii) Determine the validity of the argument :

All men are fallible

All kings are Men.

Therefore, all kings are fallible.

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