

# B. Tech Third Year : 5<sup>th</sup> Semester

## SIGNALS & SYSTEMS, DEC., 2012

(FOR 5 EC 1 BRANCH OF ENGINEERING)

**Times : 3 Hours**

**Min. Passing Marks : 24**

**Total Marks : 80**

*Instructions to Candidates : Attempt overall five questions selecting one question from each unit. All questions carry equal marks, (Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitable be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)*

### UNIT-I

1. (a) Show that if  $x_1[n]$  is an odd signal, and  $x_2[n]$  is an even signal, then  $x_1[n]x_2[n]$  is an odd signal. [8]
- (b) Let  $x(n)$  be an arbitrary signal with even and odd parts denoted by
 
$$x_e[n] = \text{Ev}\{x[n]\}$$

$$x_o[n] = \text{Od}\{x[n]\}$$

show that, 
$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] \quad [8]$$

**OR**

1. (a) Given that  $x(t) = \begin{cases} t & t > 0 \\ -2t & t < 0 \end{cases}$ 
  - (i) Sketch  $x_e(t)$
  - (ii) Sketch  $x_o(t)$  [8]
- (b) Show that the discrete time system whose input  $x(n)$  & output  $y(n)$  are related by:  $y[n] = \text{Re}\{x[n]\}$  is additive. Does this system remain additive if its input output relationship is changed to  $y(n) = \text{Re}\{e^{j\pi n/4}x[n]\}$  (Do not assume  $x(n)$  is real in this problem). [8]

### UNIT-II

2. Consider the signal  $x(n) = \alpha^n 4(n)$ 
  - (i) Sketch the signal  $g(n) = x(n) - \alpha x(n-1)$  [8]
  - (ii) Use the result of part (a) in conjunction with properties of convolution in order to determine a sequence  $h(n)$  such that  $x(n)*h(n) = (1/2)^n \{4(n+2) - 4(n-2)\}$  [8]

**OR**

2. (a) Suppose that the input to this system is

$$x[n] = \begin{cases} 0 & \text{if } h[-n] = 0 \\ \frac{h[-n]}{[h[-1]]} & \text{if } h[-n] \neq 0 \end{cases}$$

Does the input signal represent a bounded input? If so, what is the smallest number  $B$  such that  $|x[n]| < B$  for all  $n$ ? [8]

- (b) Justify that a discrete time LTI system is causal if and only if its response  $s[n]$  is zero for  $n < 0$ . [8]

### UNIT-III

3. Consider the signal

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin(k\pi/4)}{(k\pi/4)} \delta(t - k\pi/4)$$

- (a) Determine  $g(t)$  such that,  $x(t) = \frac{\sin t}{\pi t} g(t)$  [8]
- (b) Use the multiplication property of the Fourier transform

to argue that  $x(j\omega)$  is periodic specify  $x(j\omega)$  over one period. [8]

**OR**

3. (a) Suppose that  $x(t)$  is an odd-harmonic periodic signal with period 2 such that  $x(t) = t$  for  $0 < t < 1$ . Sketch  $x(t)$  and its Fourier series coefficients. [8]
- (b) Derive the convolution property  $x(n)*h(n) \leftrightarrow x(e^{j\omega})H(e^{j\omega})$  [8]

### UNIT-IV

4. (a) We are given the following facts about a real signal  $x(t)$  with laplace transform  $x(s)$ 
  - (i)  $x(s)$  has exactly two poles.
  - (ii)  $x(s)$  has no zeros in the finite  $s$  plane.
  - (iii)  $x(s)$  has a pole at  $s = -1+j$
  - (iv)  $e^{2t}x(t)$  is not absolutely integrable.
  - (v)  $x(0) = 8$
 Determine  $x(s)$  & specify its ROC. [8]
- (b) Determine the laplace transform of  $x(t) = e^{-t}$  with ROC [8]

**OR**

4. (a) Consider the linear, discrete time, shift - invariant system with input  $x(n)$  & output  $y(n)$  for which  $y(n-1) - 10/3y(n) + y(n+1) = x(n)$ . The system is stable, determine the unit sample response. [8]
- (b) Find the inverse z-transform of  $x(z) = \log(1-2z)$   $|z| < 1/2$ . By using the power-series expansion

$$\text{Log}(1-w) = -\sum_{i=1}^{\infty} \frac{w^i}{i}, |w| < 1 \quad [8]$$

### UNIT-V

5. Let  $x(t)$  be a band limited signal such that  $x(j\omega) = 0$  for  $|\omega| \geq \pi/T$ .

- (a) If  $x(t)$  is sampled using a sampling period  $T$ , determine an interpolating function  $g(t)$  such that

$$\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} x(nT)g(t-nT)$$

- (b) Is the function  $g(t)$  unique? Specify

**OR**

5. (a) Define the following terms:
  - (i) Higher frequency component of the message signal.
  - (ii) Nyquist rate
  - (iii) Sampling rate or frequency [8]
- (b) Discuss real and Natural sampling. [8]