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4E4131

Roll No. ______________

Total No. of Pages : 7

4E4131

B. Tech. IV-Sem. (Main) Exam; April-May 2017
Electronics & Communication Engg.
4EC2A Random Variables & Stochastic Processes

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 24

Instructions to Candidates :-

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*
Units of quantities used / calculated must be stated clearly.
Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. NIL 2. NIL

UNIT - I

1 (a) State the theorem of total probability and Baye's theorem on inverse probability.

8

(b) For a certain binary communication channel, the probability that a transmitted '0' is received as '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (i) a '1' is received (ii) a '1' was transmitted given that '1' was received.

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OR

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1 (a) A fair dice is rolled 5 times. Find the probability that 1 shows twice, 3 shows twice and 6 shows once. **5**

(b) State Bernoulli's theorem on independent trials. **6**

(c) Each of two persons A and B tosses 3 fair coins. What is the probability that they obtain the same number of heads ? **5**

UNIT - II

2 (a) A random variable (continuous) 'x' has a pdf $f(x) = k\,x^2 e^{-x};\ x \geq 0$. Find k, mean and variance. **8**

(b) If a continuous random variable 'x' has Rayleigh density $f(x) = \frac{x}{\alpha^2} e^{-\frac{x^2}{2\alpha^2}} X v(x)$, find $E(X^n)$ and deduce the values of $E(X)$ and $\operatorname{var}(X)$. **8**

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2 (a) If 'x' is $N\left(\mu;\sigma^2\right)$ then show that $z=\frac{(x-\mu)}{\sigma}$ is a standard normal random variable; that is $N(0',1)$. **8**

(b) A noisy transmission channel has a per digit error probability P_e=0.01.

(i) Calculate the probability of more than one error in 10 received digits.

(ii) Repeat using Poisson approximation. **5**

(c) The pdf of a random variable x is given by

$$f_x(x)=\begin{cases} k & a\le x\le b \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P((x)\le c)$ for $c=\frac{1}{2}$ if $a=-11,\ b=2$. **3**

UNIT - III

3 (a) The joint pdf of (x, y) is given by $f(x,y)=24\,xy;\ x>0,\ y>0,\ x+y\le 1$ and $f(x,y)=0$, elsewhere, find the conditional mean and variance of y given x. **8**

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(b) State central limit theorem and verify it for random variables (independent) x_k, where for each $k, P(x_k = \pm 1) = \frac{1}{2}$.

8

OR

3 (a) Calculate the correlation coefficients for the following ages of husband (x) and wives (y).

x:	23	27	28	28	29	30	31	33	35	36
y:	18	20	22	27	21	29	27	29	28	29

8

(b) Let x and y be defined by

$x = \cos \Theta$ and $y = \sin \Theta$

where Θ is a random variable uniformly distributed over $[0, 2\pi]$.

(i) Show x and y are uncorrelated

(ii) Show that x and y are not independent.

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UNIT - IV

4 (a) Consider a random process $x(t)$ is defined by

$$x(t) = u\cos t + v\sin t \quad -\infty < t < \infty$$

where 'u' and 'v' are independent random variables, each of which assumes the values -2 and 1 with the probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Show that $x(t)$ is wss but not strict sense stationary. **8**

(b) Verify the equations :

(i) $R_{xy}(-\tau) = R_{yx}(\tau)$

(ii) $\left|R_{xy}(\tau)\right| \le \sqrt{R_x(0)R_y(0)}$

(iii) $\left|R_{xy}(\tau)\right| \le \frac{1}{2}\left[R_x(0) + R_y(0)\right]$. **8**

OR

4 (a) If the input to a continuous time linear system is a random process $\{X(t),\ t \in T_x\}$ and output of the linear system is $\left(Y(t), t \in T_y\right)$.

Find the autocorrelation function of $Y(t)$. **8**

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(b) If $X(t)$ is a wss random process and has a m.s. derivative $X'(t)$ then show that

(i) $R_{XX'}(\tau) = \frac{d}{d\tau} R_x(\tau)$

(ii) $R_{X'}(\tau) = -\frac{d^2}{d\tau^2} R_x(\tau)$.

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UNIT - V

5 (a) A wss random process $X(t)$ with autocorrelation function $R_x(\tau) = \bar{e}^{a|\tau|}$ where 'a' is a real positive constant is applied to the input of an LTI system with impulse response $n(t) = \bar{e}^{bt} u(t)$ find the autocorrelation function of the output $Y(t)$.

8

(b) A zero mean wss random process is called band limited white noise if its spectral density is given by

$$S_X(w) = \begin{cases} N_0/2 & |\omega| \le \omega B \\ 0 & |\omega| > \omega B \end{cases}$$

Find the autocorrelation function of $X(t)$.

8

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OR

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5 (a) Let $Y(t)$ be the output of an LTI system with impulse response $n(t)$ when a wss random process $X(t)$ is applied as input. Show that

(i) $S_{xy}(w) = n(w) S_x(w)$

(ii) $S_y(w) = n^*(w) S_{xy}(w)$

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(b) Consider a wss process $X(t)$ with autocorrelation function $R_x(z)$ and power spectral density $S_x(w)$ let $X'(t) = dx(t) | dt$ show that $S_x(w) = w^2 S_x(w)$.

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