1E1022

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1E1022

B.Tech. I - Sem.(Main/Back) and Reback
Exam - Jan-Feb. 2012
102 - Engineering Mathematics-I
(Common to all Branches of Engg.)

Time : 3 Hours **Maximum Marks : 80**

Min. Passing Marks : 24

Instructions to Candidates:

Attempt overall ***five questions*** *selecting* ***one question*** *from* ***each unit****. All questions carry* ***equal*** *marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. Nil 2. Nil

UNIT-I

1. (a) Find the equation of the curve on which lie the points of intersection of the curve

$$x^3-5x^2y+8xy^2-4y^3+x^2-3xy+2y^2-1=0$$

and its asymptotes. 8

1. (b) Show that every point where the curve $y=c\sin\frac{x}{a}$ meets the x-axis is a point of inflexion. 8

OR

1. (a) Trace the curve: $y^2(a+x)=x^2(3a-x)$ 8

1. (b) Show that in the parabola $y^2=4ax$, the radius of curvature at any point P is $\frac{2(SP)^{\frac{3}{2}}}{\sqrt{a}}$

Where S is the focus of the parabola. Also if P_1 and P_2 are radius of curvatures at the extremities of a focal chord of the above parabola, show that:

$(P_1)^{-2/3}+(P_2)^{-2/3}=(2a)^{-2/3}$ [8]

UNIT-II

2 (a) State Euler's theorem on homogeneous functions.

Verify Euler's theorem for the function

$$f(x,y)=\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}$$

[8]

2 (b) Find the maximum value of u, where

$u=\sin x \sin y \sin(x+y)$ [8]

OR

2 (a) ABC is an acute angled triangle with fixed base BC. If δb, $\delta c, \delta A$ *and* δB are small increments in b, c, A and B respectively when the vertex A is given a small displacement parallel to BC. Prove that

$$\delta A = h\delta x\left(\frac{1}{c^2}-\frac{1}{b^2}\right)$$

Where h is the unaltered height of the triangle. [8]

(b) A rectangular box, open at the top, is to have a volume of 32 cubic meters. Find its dimensions so that total surface is minimum. [8]

UNIT-III

3 (a) Prove that the surface and volume of the solid generated by revolving the loop of the curve $x=t^2$, $y=t-\frac{t^3}{3}$ about the X-axis are respectively

$$3\pi \text{ and } \frac{3\pi}{4}$$

[8]

(b) Prove that: $\int_0^1 \sqrt{1-x^4}\,dx = \frac{\left\{\sqrt{\frac{1}{4}}\right\}^2}{6\sqrt{(2\pi)}}$ [8]

OR

3 (a) Prove that the whole length of the curve $x^2(a^2-x^2)=8a^2y^2$ is $\pi a\sqrt{2}$ [8]

3 (b) Evaluate $\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$ [8]

By changing the order of integration.

UNIT-IV

4. Solve:

(a) $\frac{dy}{dx} = \frac{2x-y+1}{x+2y-3}$ [5]

(b) $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$ [5]

(c) $(D-1)^2(D^2+1)^2y=\sin^2\frac{x}{2}\,e^x$ [6]

OR

4 Solve:

(a) $\frac{dy}{dx}=\frac{1}{xy(x^2y^2+1)}$ [5]

(b) $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ [5]

(c) $(D^2-3D+2)y=\sin 3x+x^2+x+e^{4x}$ [6]

UNIT-V

5. Solve:

(a) $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x)$ [5]

(b) $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-3)y=e^{x^2}$ [5]

(c) $\sin^2x\,\frac{d^2y}{dx^2}=2y$ [6]

OR

5 (a) $(1+x^2)\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx}+4y=0$ [8]

(b) Use the method of variation of parameters to solve:

$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$ [8]