

2E2002

Roll No. 15EEBEC034

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B.Tech. II Semester (Main/Back) Examination, June/July - 2016
202 Engg. Mathematics - II

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 26

Instructions to Candidates:

Attempt any five questions, selecting one question from each unit. All questions carry equal marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)

Unit - I

1. a) A plane passes through a fixed point (a,b,c) and cut the axis in A,B,C. Show that the locus of the centre of the sphere OABC is (8)

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

- b) Two spheres of radii r_1 and r_2 cut orthogonally, prove the radius of their common circle is (8)

$$\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

OR

1. a) Define right circular cone. Find the equation of the right circular cone whose vertex is origin, axis is x - axis and semi vertical angle is α . (2+6=8)
b) Define right circular cylinder. Find the equation of a right circular cylinder whose axis is

$$\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{3}$$

and which passes through (0,0,1)

(2+6=8)

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(1)

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Unit - II

2. a) Find the rank of the following matrix by reducing it to the normal form :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad (8)$$

- b) Test the consistency of the following system of equations and if possible solve it:

$$\begin{aligned} 2x - 3y + 7z &= 5 \\ 3x + y - 3z &= 13 \\ 3x + 19y - 47z &= 32 \end{aligned} \quad (8)$$

OR

2. a) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix} \quad (8)$$

- b) State Cayley Hamilton Theorem, verify it for the matrix.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \text{ and find } A^{-1}. \quad (2+6=8)$$

Unit - III

3. a) A particle moves on the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$, where t denote time. Find the components of velocity and acceleration at $t = 1$ in the direction of vector $\hat{i} - 3\hat{j} + 2\hat{k}$. (8)

- b) Prove that :

i) $\nabla^2(r^n) = n(n+2)r^{n-2}$, if $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

ii) $\text{Curl}(\vec{a} \times \vec{r}) = 2\vec{a}$, if \vec{a} is a constant vector. (4+4=8)

OR

3. a) If \vec{a} and \vec{b} are differentiable vector functions, then show that :

i) $\text{div}(\vec{a} \times \vec{b}) = \vec{b} \cdot \text{curl} \vec{a} - \vec{a} \cdot \text{curl} \vec{b}$

ii) $\text{div} \text{curl} \vec{a} = 0$

(5+3=8)

b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, curve c is rectangle in the xy-plane bounded by $x = 0$; $x = a$; $y = 0$; $y = b$. (8)

Unit - IV

4. a) Evaluate $\iiint_V \vec{F} \cdot \hat{n} ds$ by using Gauss's divergence theorem for $\vec{F} = xy\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron $x = y = z = 0$, $x + y + z = 1$. (8)

b) State stoke's theorem. Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2dy]$, where c is the closed curve of the region bounded by $y = x^2$ and $y = x$. (2+6=8)

OR

a) Obtain the Fourier series for the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the following :

i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$

ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$

(5+3=8)

b) Express $f(x)$ in a fourier series upto the second harmonic for the following data :

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x) :	1.98	2.15	2.77	-0.22	-0.31	1.43	1.98

(8)

Unit - V

5. a) Solve the following differential equation in series.

$$\frac{d^2y}{dx^2} + x^2y = 0 \quad (8)$$

- b) Solve:

$$x(y^2 - z^2)q - y(z^2 + x^2)q = z(x^2 + y^2) \quad (8)$$

OR

5. a) Solve:

$$x^2p^2 + y^2q^2 = z^2 \quad (8)$$

- b) Find a complete integral of

$$q = (z + px)^2$$

by using charpit's method. (8)

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