2E2002

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B. Tech. II Sem. (Back) Exam., May – 2018

202Engineering Mathematics - II

Time: 3 Hours **Maximum Marks: 80**
Min. Passing Marks: 24

Instructions to Candidates:

*Attempt any **five questions,** selecting **one question** from **each unit.** All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

Units of quantities used/calculated must be stated clearly.

Use *of following supporting material is permitted during examination. (Mentioned in form No.205)*

1. NIL 2. NIL

UNIT-I

Q.1 (a) A plane passes through the fixed point (a, b, c) and cut the axes in A, B, C. Show that the locus of the centre of the sphere OABC is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$ [8]

(b) Find the equation of the right circular cylinder whose guiding curve is the circle

$$x^2+y^2+z^2=9, \quad x-y+z=3$$ [8]

OR

Q.1 (a) Find the equation of the sphere that passes through the circle $x^2+y^2+z^2-2x+3y-4z+6=0$, $3x-4y+5z=15$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally. [8]

(b) Find the equation of the right circular cone with vertex at the origin, axis is the line $\frac{x}{2} = \frac{y}{-4} = \frac{z}{3}$ and which passes through the point (1, 1, 2). [8]

UNIT-II

Q.2 (a) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have (i) no solution, (ii) unique solution and (iii) an infinite number of solution [8]

(b) Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

Hence, reduce the given matrix into the diagonal form. [8]

OR

Q.2 (a) Test the consistency of the following equations, and if possible, find the solution:

$5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ [8]

(b) **State Cayley – Hamilton theorem.** Verify it for the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Hence find A^{-1}. [8]

UNIT-III

Q.3 (a) If $\vec{F} = (y^2 + 2xz^2)\hat{i} + (2xy - z)\hat{j} + (2x^2z - y + 2z)\hat{k}$ be a vector point function, then show that $\vec{F}$ is irrotational and hence find its scalar potential. [8]

(b) Evaluate $\iint_s \left(yz\hat{i} + zx\hat{j} + xy\hat{k}\right).ds$, where s is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. [8]

OR

Q.3 (a) Show that $\vec{F} = \left(2xy + z^3\right)\hat{i} + x^2\hat{j} + 3z^2x\hat{k}$ is a conservative field. Find its scalar potential and also the work done in moving a particle from (1, -2, 1) to (3, 1, 4). [8]

(b) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2). [8]

UNIT-IV

Q.4 (a) Verify Green's theorem in a plane for, $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where c is the boundary of the region defined by the lines x = 0, y = 0 and x + y = 1. [8]

(b) Obtain the Fourier series for the function $f(x) = x^2$, $-\pi < x < \pi$ and deduce from it

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots$$

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots\ldots$$

and $$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\ldots\ldots$$ [8]

OR

Q.4 (a) Use Stoke's theorem to evaluate $\int_c \vec{F}.d\vec{r}$, where $\vec{F} = (\sin x - y)\hat{i} - \cos x\,\hat{j}$ and c is the boundary of the triangle whose vertices are (0, 0), $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. [8]

(b) Obtain the first three cosine terms and constant terms in the Fourier series for y, where [8]

x	0	1	2	3	4	5
y	4	8	15	7	6	2

UNIT-V

Q.5 (a) Solve: $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ [6]

(b) Find the complete integral of $p^3 + q^3 = 3pqz$ [6]

(c) Find the singular integral of $z = px + qy + p^2 + q^2$ [4]

OR

(a) Solve in series: $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0$ [8]

(b) Use Charpit's method to solve:

$px + qy = pq$ [8]
