Roll No. ________

Total No of Pages: 3

1E2401

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B. Tech. I - Sem. (Main/Back) Exam., Dec. 2019

1FY2-01 Engineering Mathematics-I

Time: 3 Hours

Maximum Marks: 160
Min. Passing Marks: 56

Instructions to Candidates:

Attempt all ten questions from Part A, fiver questions out of seven questions from Part B and four questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART – A

(Answer should be given up to 25 words only) **[10×3=30]**

All questions are compulsory

Q.1 Evaluate $\int_0^\infty e^{-x^2}dx$

Q.2 Find the volume generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x – axis.

Q.3 Write the condition for p series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ to be convergent and divergent.

Q.4 Determine the radii of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} .z^n$

Q.5 State the Parseval's theorem.

Q.6 Evaluate $\lim_{(x,y)\to(0,0)} \frac{2x^3 - y^3}{x^2 + y^2}$.

Q.7 If $f(x, y, z) = 2x^2y - y^3 z^2$, then find the grad f at the point (1, -2, -1).

Q.8 Evaluate $\int_0^1 \int_0^2 dx\, dy$

Q.9 Write the coordinate of center of gravity of a solid.

Q.10 Write the statement of Gauss's divergence theorem.

PART – B

(Analytical/Problem solving questions) **[5×10=50]**

Attempt any four questions

Q.1 Prove that $\int_0^\infty \frac{dx}{1+x^4} = \frac{\pi}{2\sqrt{2}}$

Q.2 Use Taylor's theorem to show that –

$$\sin(x+h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x - \frac{h^3}{3!}\cos x + \cdots$$

Q.3 Find the half range cosine series of $f(x) = x(\pi - x)$ in the interval $(0, \pi)$.

Q.4 If $x^x y^y z^z = c$, then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x\, \partial y} = -(x \log ex)^{-1}$

Q.5 Show that curl grad $r^n = 0$; where $\vec{r} = xi + yj + zk$

Q.6 Change the order of integration and evaluate $\int_0^1 \int_{e^x}^{e} \left[\frac{1}{\log y}\right] dx\, dy$

Q.7 Use Gauss theorem to evaluate $\int_S F.n\, ds$, where $F = 4xy\, i + yz\, j - xz\, k$ and S is the surface of the cube bounded by the planes x = 0, x = 2, y = 0, y = 2, z = 0, z = 2.

PART – C

(Descriptive/Analytical/Problem Solving/Design Questions) [4×20=80]

Attempt any two questions

Q.1 Prove that the surface area of the solid generated by revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis is –

$2\pi ab\left[\sqrt{1-e^2} + \frac{1}{e}(\sin^{-1}e)\right]$, where $b^2 = a^2(1-e^2)$.

Q.2 Test the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{1^2.5^2.9^2 --- (4n-3)^2}{4^2.8^2.12^2 --- (4n)^2}$$

Q.3 Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$. Hence, using Parseval's theorem, prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

Q.4 Prove that the extreme value of the function $u = a^2x^2 + b^2y^2 + c^2z^2$, where $x^2 + y^2 + z^2 = 1$ and $\ell x + my + nz = 0$, given by the equation.

$$\frac{\ell^2}{u-a^2} + \frac{m^2}{u-b^2} + \frac{n^2}{u-c^2} = 0$$

Q.5 Verify the Green's theorem is plane for $\int_C \left[(3x^2 - 8y^2)dx + (4y - 6xy)dy\right]$, where C is the boundary of the region defined by $x = 0$, $y = 0$, and $x + y = 1$.
